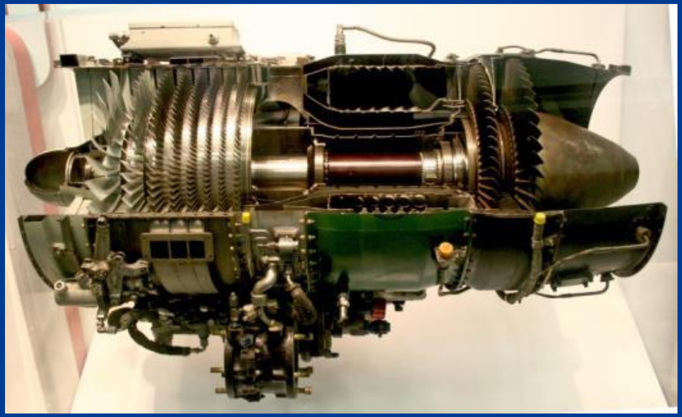
Applied Thermodynamics - II



Gas Turbines – Shaft Power Real Cycles

Sudheer Siddapureddy

sudheer@iitp.ac.in

भारतीय प्रौद्योगिकी संस्थान पटना

INDIAN INSTITUTE OF TECHNOLOGY PATNA

Department of Mechanical Engineering

Real Cycles: Differ from Ideal Cycles

1. High velocity in turbomachinery: ΔKE cannot be ignored.
2. Compression and expansion are irreversible adiabatic processes: entropy increases.
3. Fluid friction: pressure losses in CC, HE, inlet and outlet ducts.
4. No perfect heat exchange keeping the size in mind.
5. Mass flow is assumed to be the same. Bleeding from compressor to cool turbine (1 to 2%) is compensated by the addition of fuel.
6. C_p and γ vary throughout due to changes of T, chemical composition.
7. More work for compression due to bearing, 'windage' friction in the transmission between compressor and turbine, and to drive ancillary components such as fuel and oil pumps.
8. η for ideal cycle is unambiguous. Open cycle with internal combustion? Express the performance in terms of fuel consumption per unit net work output, Specific Fuel Consumption (*SFC*).

1. Change in Fluid Velocity

Stagnation Properties

- KE terms in the steady flow energy equation can be accounted for, implicitly by making use of stagnation enthalpy.
- h_0 is the enthalpy which a gas stream of h and c would posses when brought to rest adiabatically and without work transfer.

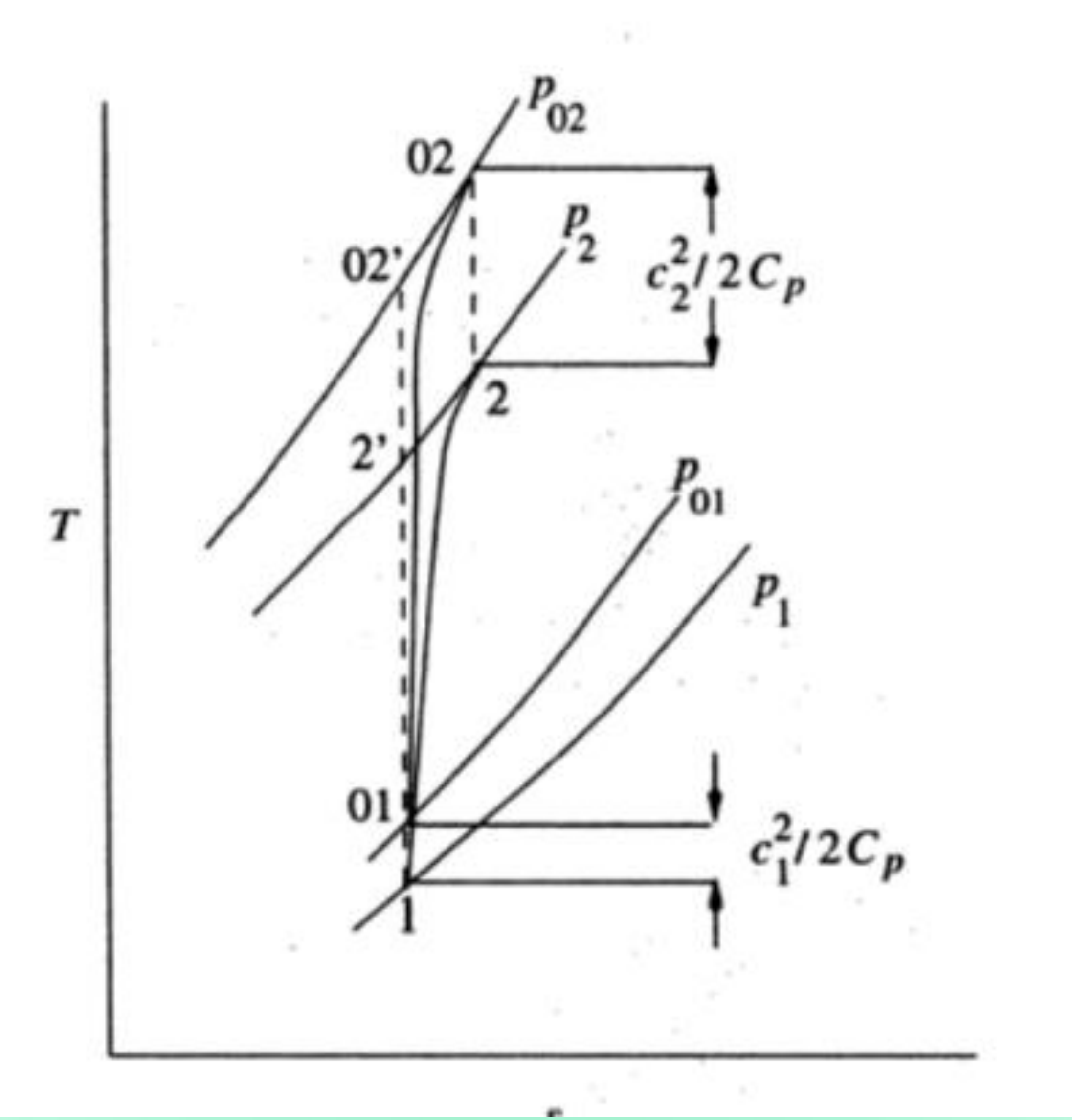
$$(h_0 - h) + \frac{1}{2}(0 - c^2) = 0$$

$$h_0 = h + \frac{c^2}{2}$$

$$T_0 = T + \frac{c^2}{2C_p}$$

static **dynamic temperature**

Stagnation Properties: Compression



1. Change in Fluid Velocity

Adiabatic compression, the energy equation becomes:

$$W = -C_p \left[\underbrace{(T_2 - T_1)}_{\text{static component}} + \underbrace{\left(\frac{c_2^2}{2C_p} - \frac{c_1^2}{2C_p} \right)}_{\text{dynamic component}} \right]$$

$$W = \underbrace{-C_p(T_{02} - T_{01})}_{\text{stagnation}}$$

Similarly for a heating process without work transfer

$$Q = C_p(T_{02} - T_{01})$$

1. Change in Fluid Velocity

When a gas is brought to rest, adiabatically and reversibly:

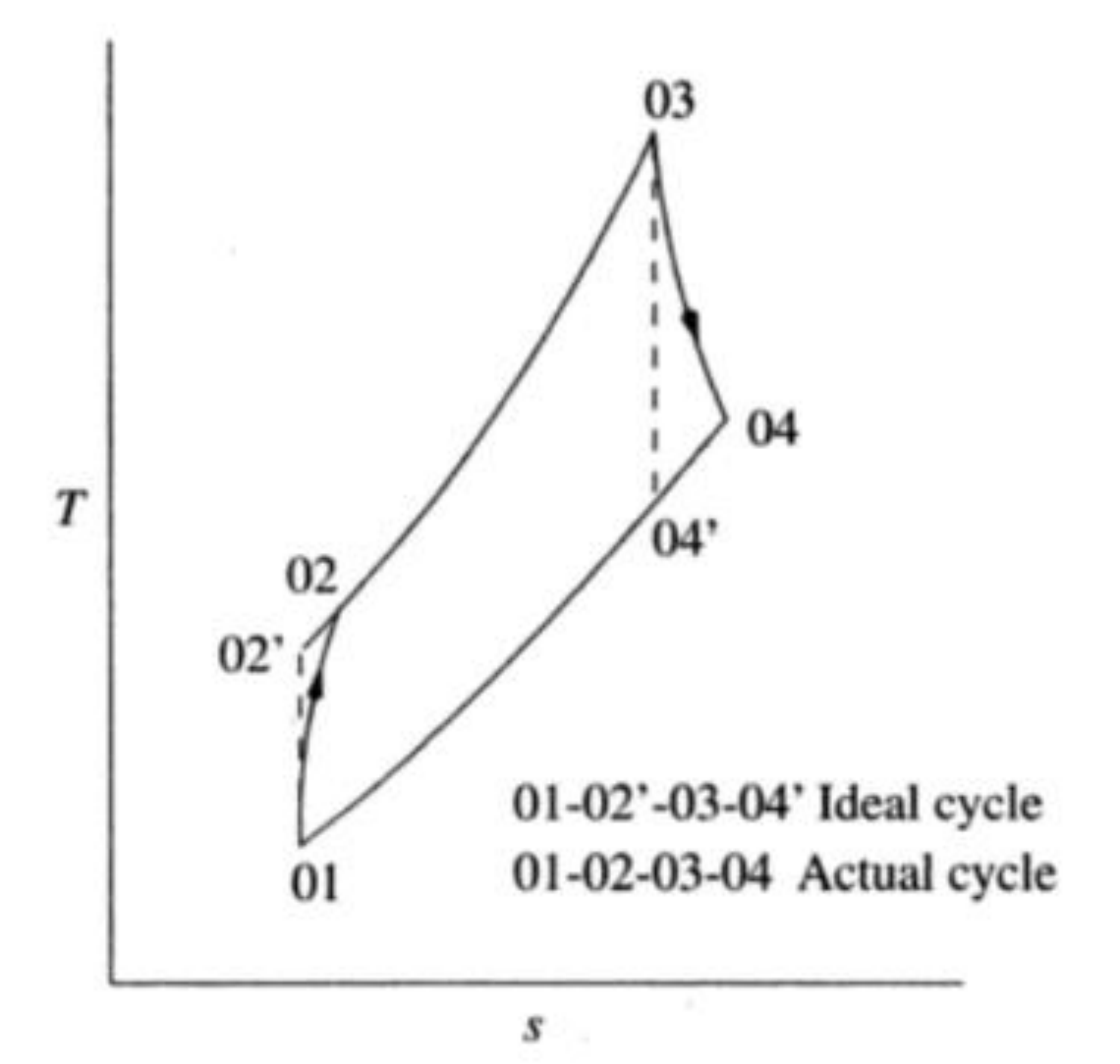
$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

The effect of the inclusion of velocity in the calculation: Refer to stagnation conditions (h_0, T_0, p_0) but not static conditions (h, T, p) as we did in the ideal cycle analysis.

Real Cycles: Differ from Ideal Cycles

1. ~~High velocity in turbomachinery: ΔKE cannot be ignored.~~
2. Compression and expansion are irreversible adiabatic processes: entropy increases.
3. Fluid friction: pressure losses in CC, HE, inlet and outlet ducts.
4. No perfect heat exchange keeping the size in mind.
5. Mass flow is assumed to be the same. Bleeding from compressor to cool turbine (1 to 2%) is compensated by the addition of fuel.
6. C_p and γ vary throughout due to changes of T, chemical composition.
7. More work for compression due to bearing, 'windage' friction in the transmission between compressor and turbine, and to drive ancillary components such as fuel and oil pumps.
8. η for ideal cycle is unambiguous. Open cycle with internal combustion? Express the performance in terms of fuel consumption per unit net work output, Specific Fuel Consumption (*SFC*).

2. Compressor and Turbine Efficiency



Compressor Isentropic Efficiency

$$\eta_c = \frac{\text{Work required for isentropic compression}}{\text{Actual compression work input}}$$

$$= \frac{h_{02'} - h_{01}}{h_{02} - h_{01}} = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}}$$

$$T_{02} - T_{01} = \frac{T_{01}}{\eta_c}\left[r_c^{(\gamma-1)/\gamma} - 1\right]$$

$$W_{c_{act}} = \frac{C_p T_{01}}{\eta_c}\left[r_c^{(\gamma-1)/\gamma} - 1\right]$$

$$r_c = \left(\frac{T_{02'}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

Turbine Isentropic Efficiency

$$\eta_t = \frac{\text{Actual turbine work output}}{\text{Isentropic turbine work output}}$$

$$= \frac{h_{03} - h_{04}}{h_{03} - h_{04'}} = \frac{T_{03} - T_{04}}{T_{03} - T_{04'}}$$

$$T_{03} - T_{04} = \eta_t T_{03}\left[1 - \frac{1}{r_t^{(\gamma-1)/\gamma}}\right]$$

$$W_{t_{act}} = \eta_t C_p T_{03}\left[1 - \frac{1}{r_t^{(\gamma-1)/\gamma}}\right]$$

3. Pressure or Flow Losses

Losses due to friction & turbulence occurs throughout the whole plant.

1. Air side intercooler loss
2. Air side heat exchanger loss
3. Combustion chamber loss (both main and reheat)
4. Gas side heat exchanger loss
5. Duct losses between components and at intake and exhaust

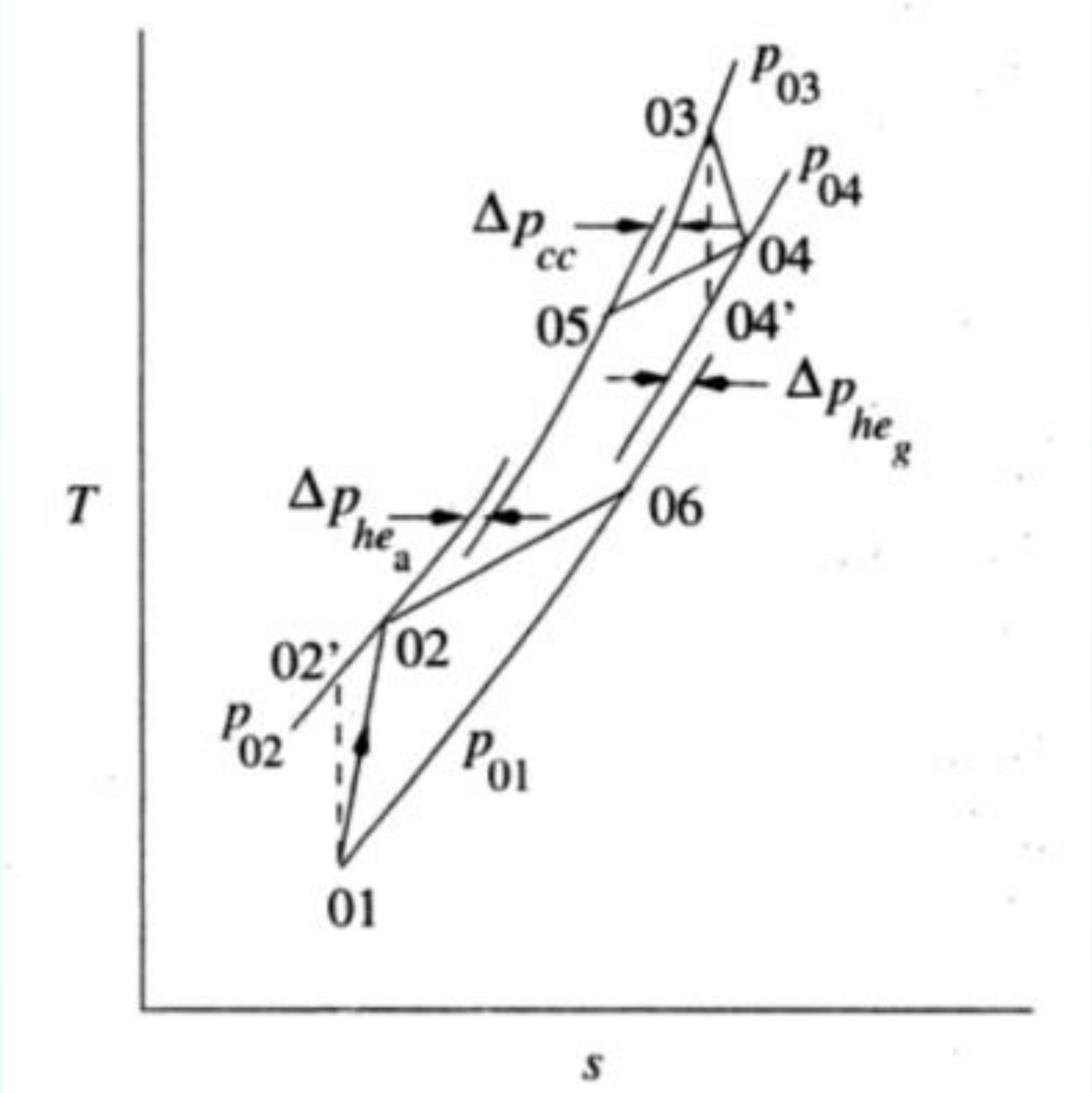
The pressure losses have the effect of decreasing r_t relative to r_c

Reduces net work output from the plant

Gas turbine cycle is very sensitive to irreversibilities:

Δp significantly effects the cycle performance

Pressure or Flow Losses



Pressure or Flow Losses

$$p_{03} = p_{02} - \Delta p_{he_a} - \Delta p_{cc}$$

$$p_{04} = p_a + \Delta p_{he_g}$$

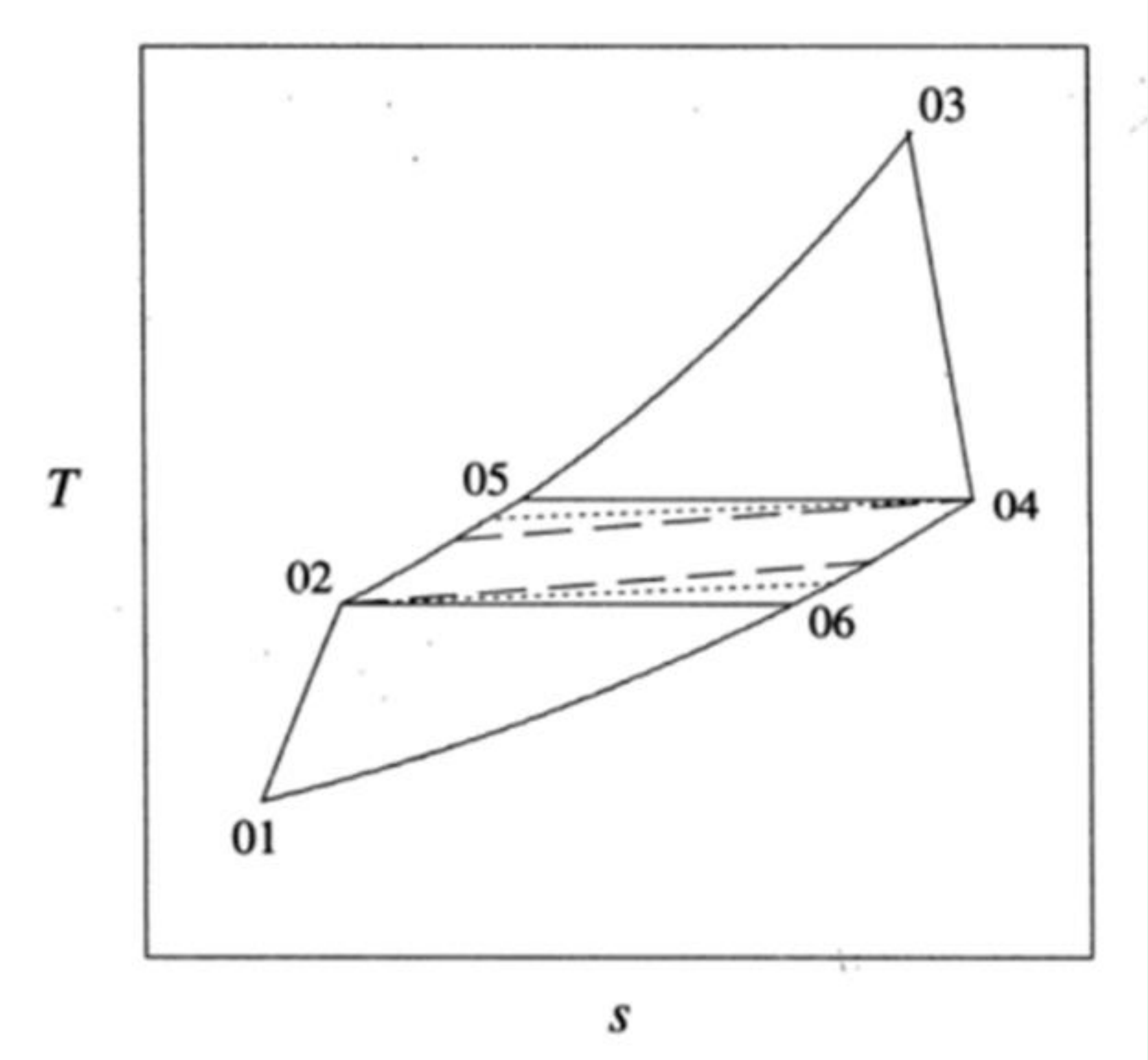
The turbine inlet pressure is then found from:

$$p_{03} = p_{02}\left(1 - \frac{\Delta p_{he_a}}{p_{02}} - \frac{\Delta p_{cc}}{p_{02}}\right)$$

Pressure losses minimized by large CC with consequent low velocities.

Typically $\frac{\Delta p_{cc}}{p_{02}} \approx 2$ to 3% for a large industrial unit and 6 to 8% for an aero engine

4. Heat Exchanger Effectiveness



4. Heat Exchanger Effectiveness

Heat transfer with small dT requires infinite area

$$\dot{m}_a C_{p_a}(T_{05} - T_{02}) = \dot{m}_g C_{p_g}(T_{04} - T_{06})$$

$$T_{05} - T_{02} = \frac{\dot{m}_g C_{p_g}}{\dot{m}_a C_{p_a}}(T_{04} - T_{06}) > T_{04} - T_{06}$$

$$\epsilon = \frac{T_{05} - T_{02}}{T_{04} - T_{02}}$$

Ratio of the temperature rise of the air $(T_{05} - T_{02})$ to the max. temperature difference available $(T_{04} - T_{02})$.

Larger areas ⇒ larger ϵ

5. Effect of Varying Mass Flow

$$\dot{m}_g = \dot{m}_a + \dot{m}_f$$

$$\frac{\dot{m}_g}{\dot{m}_a} = 1 + \frac{\dot{m}_f}{\dot{m}_a} = 1 + f$$

However, this increase is compensated by the bleeding compressed air to turbine cooling.

Overall air-fuel ratio of the order of 60:1 to 100:1.

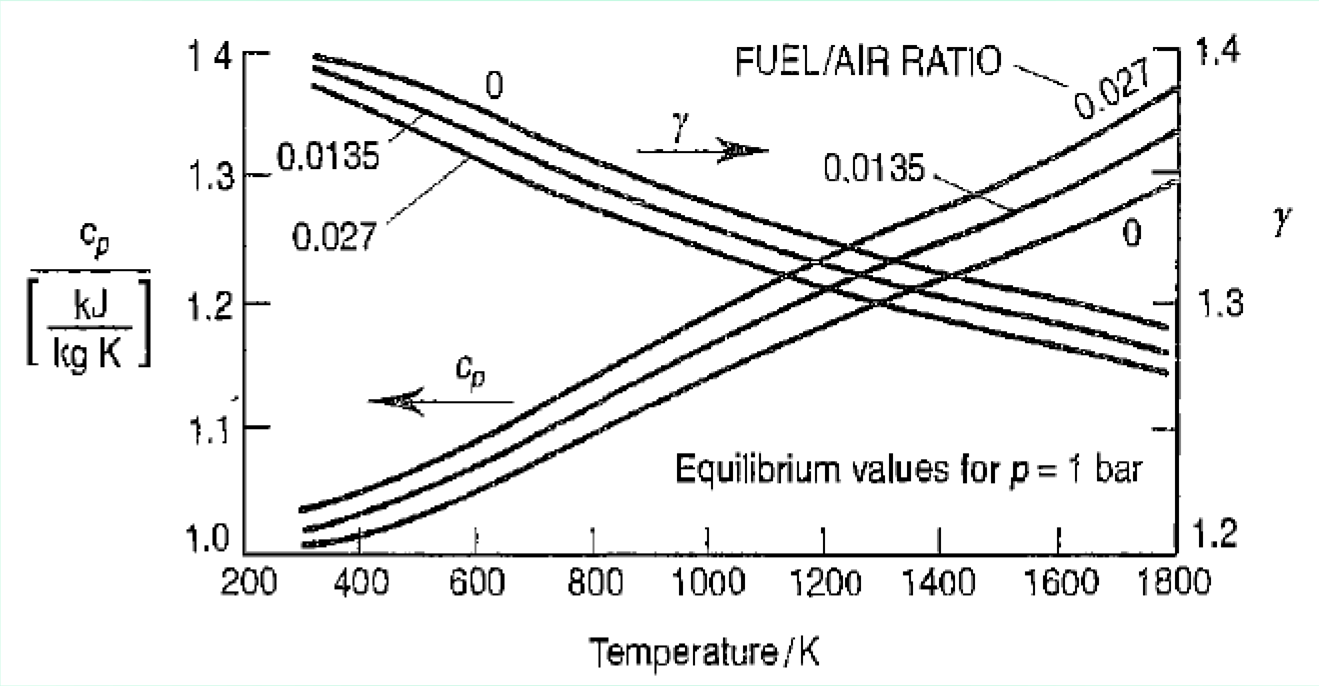
6. Effect of Variable Specific Heat

C_{p_a} is independent of P within the operating limits but varies with T

$C_{p_{300\text{K}}} = 1.005$ kJ/kg K $\qquad C_{p_{1000\text{K}}} = 1.140$ kJ/kg K

An increase of about 13.4%

However, γ decreases with T

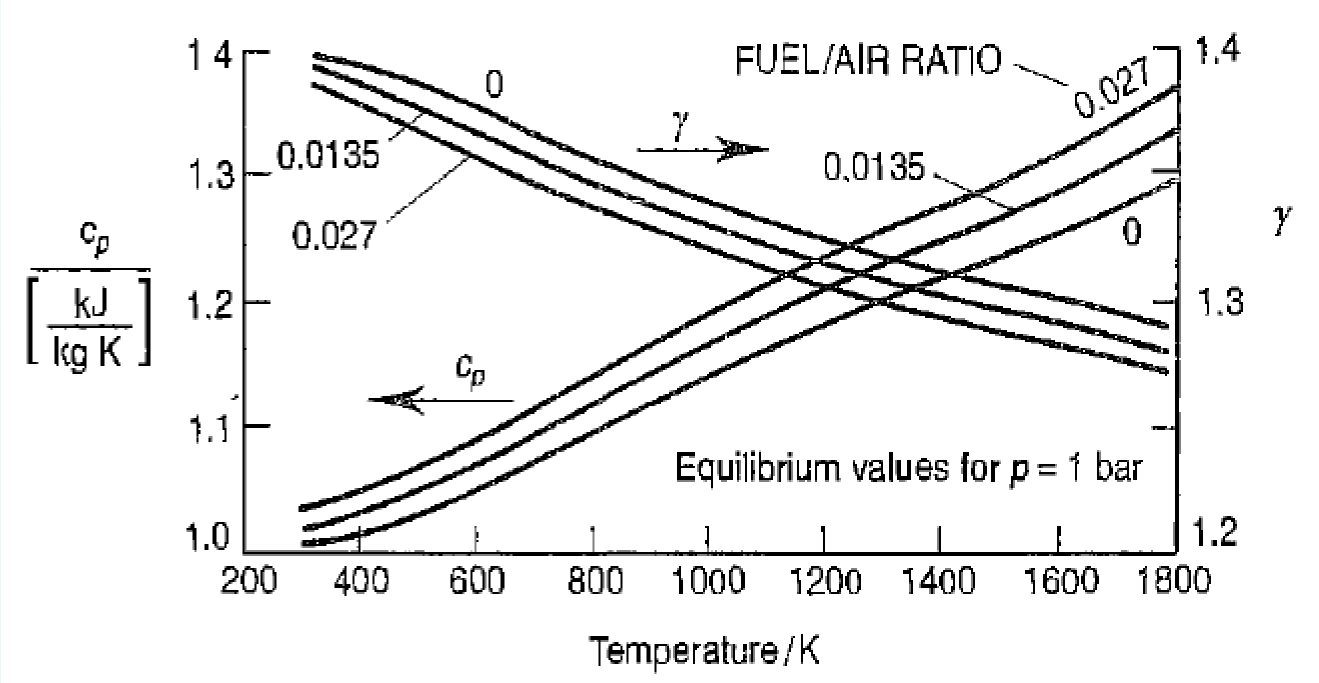


6. Effect of Variable Specific Heat

Recommended: a C_{pa} for compressor & C_{pg} for heating and expansion

$$C_{p_a} = 1.005 \text{ kJ/kg K and } \gamma = 1.40 \text{ during compression}$$

$$C_{p_g} = 1.140 \text{ kJ/kg K and } \gamma = 1.33 \text{ during heating and expansion}$$



7. Mechanical Losses

Bearing friction, windage amounts to 10% of compressor work (W_c)

$$W_c = \frac{1}{\eta_{mech}} C_{p_a}(T_{02} - T_{01})$$

8. Loss due to Incomplete Combustion

$$\eta_b = \frac{\text{Enthalpy released}}{\text{Available entalpy of fuel}} = 0.98$$

Air-fuel Ratio

$$\dot{m}_f \eta_b \Delta H_{rp} = (\dot{m}_f + \dot{m}_a) C_{pg} (T_{03} - T_{02})$$

ΔH_{rp} is the enthalpy of reaction for unit mass at 25°C

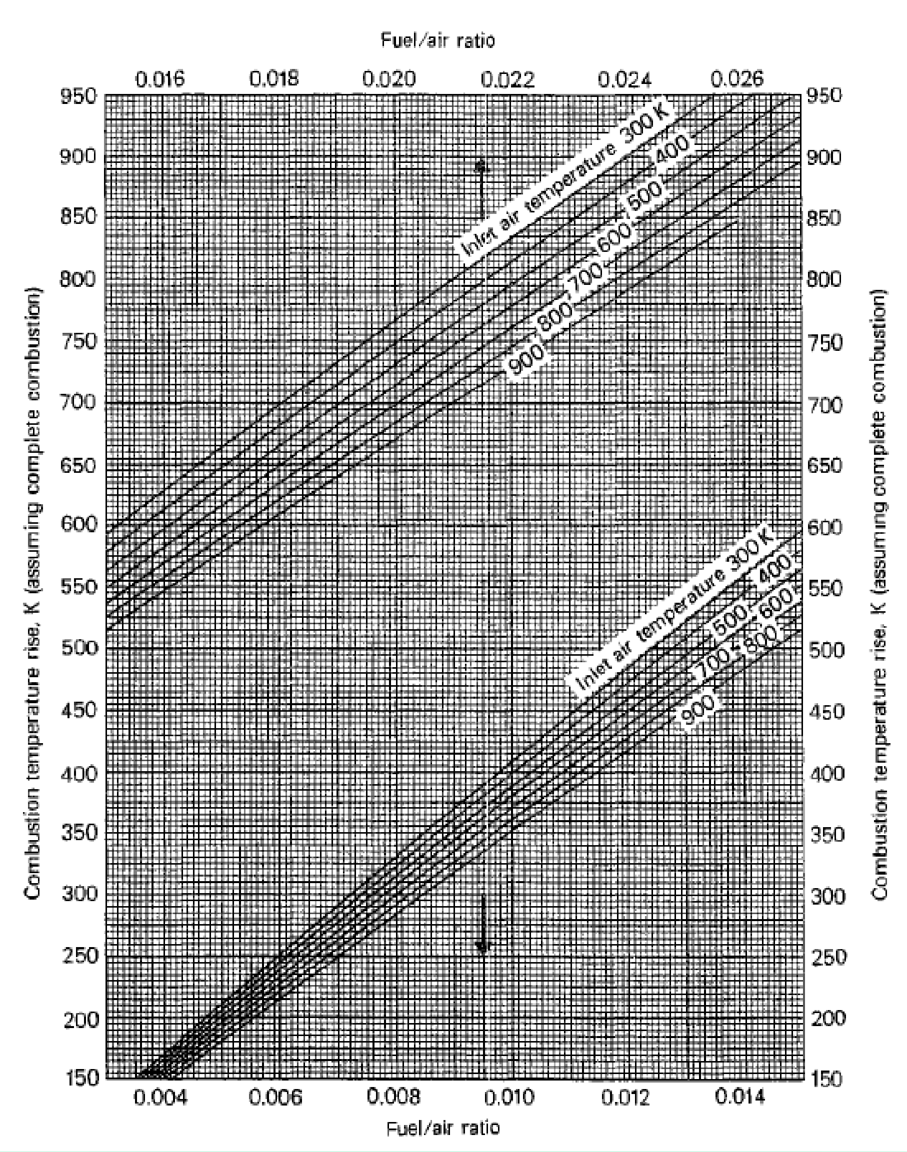
Lower calorific value

$$f \, \eta_b \, \Delta H_{rp} = C_{pg} (T_{03} - T_{02})$$

$$C_{pg} = f(T, \text{species concentration})$$

Iteratively solve for f and C_{pg}

Air-fuel Ratio



Specific Fuel Consumption

$$SFC = \frac{f}{W_N}$$

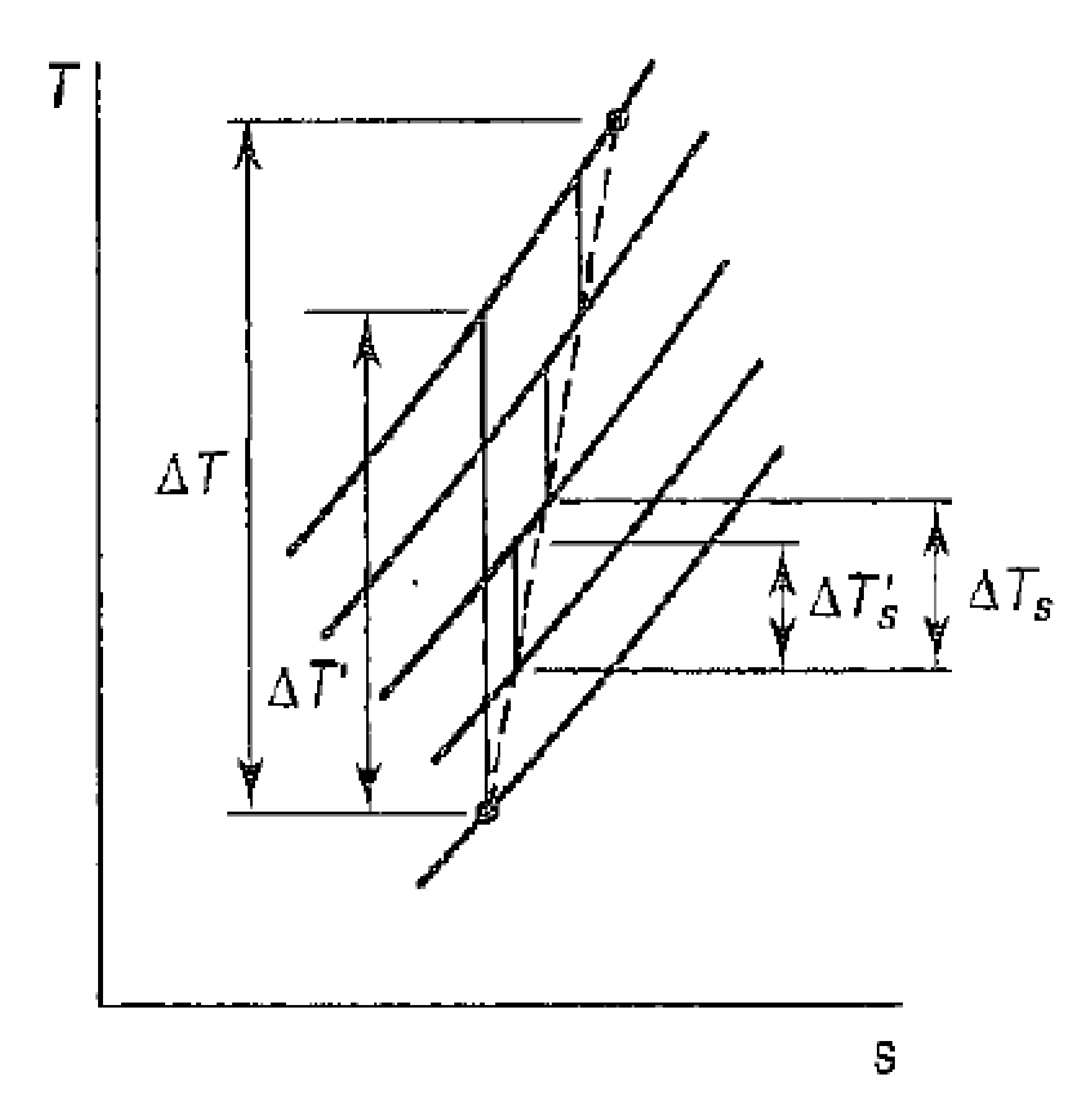
$$SFC = \frac{3600\, f}{W_N}\ \text{kg/kW h}$$

$$\eta = \frac{W_N}{f\ CV} = \frac{3600}{SFC\ CV}$$

$$CV = 43\ \text{MJ/kg}$$

Polytropic Efficiency

η_c decrease and η_t increase as the pressure ratio for which the compressor and turbine are designed increases.



Polytropic Efficiency

$$\frac{\eta_s}{\eta_c} = \frac{\sum \Delta T_s'}{\Delta T'}$$

- $\eta_s > \eta_c$ and this difference increases with number of stages (r).
- Meaning work is required due to 'preheat' effect.
- Similarly for a turbine $\eta_t > \eta_s$. Frictional 'reheating' in one stage is partially recovered as work in the next.
- Polytropic efficiency is the isentropic efficiency of an elemental stage in the process such that it is constant throughout the whole process.

$$\eta_{sc} = \frac{dT'}{dT} = \text{constant}$$

$$\frac{T}{p^{(\gamma-1)/\gamma}} = \text{constant}$$

Polytropic Efficiency

$$\frac{dT'}{T} = \frac{\gamma - 1}{\gamma}\frac{dp}{p}$$

$$\eta_{sc} = \frac{\ln(p_2/p_1)^{(\gamma-1)/\gamma}}{\ln(T_2/T_1)}$$

η_{sc}computed for given values

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma\eta_{sc}}$$

$$\eta_c = \frac{(p_2/p_1)^{(\gamma-1)/\gamma} - 1}{(p_2/p_1)^{(\gamma-1)/\gamma\eta_{sc}} - 1}$$

Polytropic Efficiency

$$T_{02} - T_{01} = T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{(n-1)/n} - 1\right]$$

$$(n-1)/n = (\gamma - 1)/\gamma\eta_{sc}$$

For Turbine, $\eta_{st} = \frac{dT}{dT'} = \text{constant}$

$$T_{03} - T_{04} = T_{03}\left[1 - \frac{1}{\left(\frac{p_{02}}{p_{01}}\right)^{(n-1)/n}}\right]$$

$$(n-1)/n = \eta_{st}(\gamma - 1)/\gamma$$

Polytropic Efficiency

For Turbine, $\eta_{st} = \frac{dT}{dT'} = \text{constant}$

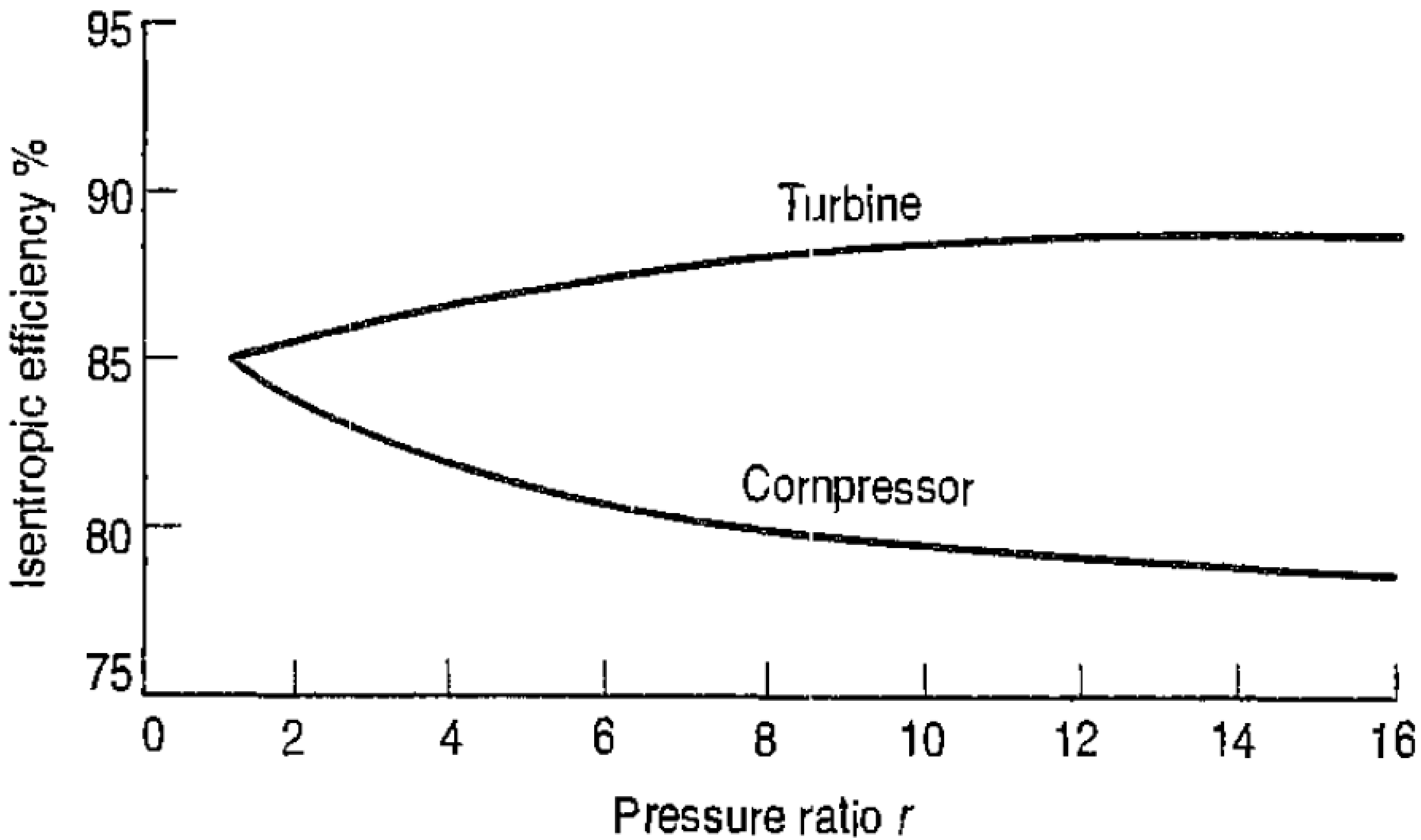
$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\eta_{st}(\gamma-1)/\gamma}$$

$$\eta_c = \frac{(p_2/p_1)^{(\gamma-1)/\gamma} - 1}{(p_2/p_1)^{\eta_{st}(\gamma-1)/\gamma} - 1}$$

$$T_{03} - T_{04} = T_{03}\left[1 - \frac{1}{\left(\frac{p_{02}}{p_{01}}\right)^{(n-1)/n}}\right]$$

$$(n-1)/n = \eta_{st}(\gamma-1)/\gamma$$

Polytropic Efficiency



Problem: Real Cycle Analysis

An oil gas turbine installation consists of a compressor, a combustion chamber and turbine. The air taken in at a pressure of 1 bar and 30°C is compressed to 6 bar, with an isentropic efficiency of 87%. Heat is added by the combustion of fuel in combustion chamber to raise the temperature to 700°C. The efficiency of the turbine is 85%. The calorific value of the oil used is 43.1 MJ/kg. Calculate for an air flow of 80 kg/min

1. the air/fuel ratio of the turbine gases
2. the final temperature of exhaust gases
3. the net power of installation
4. The overall thermal efficiency of the installation

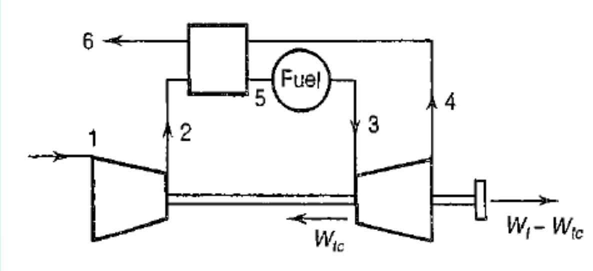
Assume C_{pa} = 1.005 kJ/kg K, γ_a = 1.4, C_{pg} = 1.147 kJ/kg K, γ_g = 1.33.

Ans: 86 kg/kg, 676.2 K, 147.2 kW, 22%

Problem: Real Cycle Analysis

Determine the specific work output, *SFC*, η for a heat-exchange cycle with the following specifications:

Compressor pressure ratio	4.0
Turbine inlet temperature	1100 K
Isentropic efficiency of compressor, η_c	0.85
Isentropic efficiency of turbine, η_t	0.87
Mechanical transmission efficiency, η_m	0.99
Combustion efficiency, η_b	0.99
Heat-exchanger effectiveness, ϵ	0.80
Pressure losses:	
Combustion chamber, Δp_{cc}	2% comp. deliv. press.
Heat-exchanger air-side, Δp_{ha}	3% comp. deliv. press.
Heat-exchanger gas-side, Δp_{hg}	0.04 bar
Ambient conditions, p_a, T_a	1 bar, 288 K

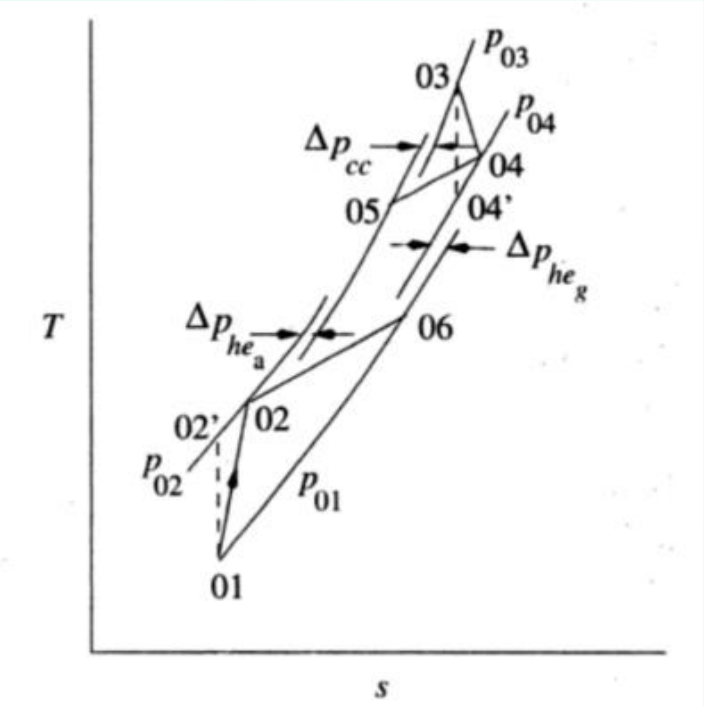


$$C_{p_a} = 1.005 \text{ kJ/kg K}, \gamma_a = 1.40; \; C_{p_g} = 1.140 \text{ kJ/kg K}, \gamma_g = 1.33$$

Problem: Real Cycle Analysis

Hint

- Compute p_{02}, T_{02} at compressor exit
- Compute W_c (using η_{mech})
- Compute p_{03}, p_{04} using Δp values
- Compute T_{04}, W_t, then W_{Nt}
- Using ϵ, find T_{05}
- Compute W_N
- Using the chart, find f corresponding to T_{03} - T_{05}
- Compute SFC, η



Ans: 137 kJ/kg, 0.253 kg/kW h, 33.1%

Problem: Real Cycle Analysis

At design speed the following data apply to a gas turbine set employing a separate power turbine, heat exchanger, reheater and intercooler between two-stage compression. Find the net power output, SFC and overall thermal efficiency.

C_{pa} = 1.005 kJ/kg K, γ_a = 1.4,
C_{pg} = 1.147 kJ/kg K, γ_g = 1.33.
Calorific value of fuel is 42 MJ/kg

Pressure loss in each side of heat exchanger is 0.1 bar.

Efficiency of compression in each stage	:	80%
Isentropic efficiency of compressor turbine	:	87%
Isentropic efficiency of power turbine	:	80%
Transmission efficiency	:	99%
Pressure ratio in each stage of compression	:	2:1
Pressure loss in intercooler	:	0.07 bar
Temperature after intercooling	:	300 K
Thermal ratio of heat exchanger	:	0.75
Pressure loss in combustion chamber	:	0.15 bar
Combustion efficiency of reheater	:	98%
Maximum cycle temperature	:	1000 K
Temperature after reheating	:	1000 K
Air mass flow	:	25 kg/s
Ambient air temperature	:	15°C
Ambient air pressure	:	1 bar